Governed Relational Geometry

A Formal Model for Evidence, Context and Valid Transformation

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This manuscript specifies public semantics and proof obligations. It does not prescribe or disclose an implementation-specific coordinate representation, metric, algorithm or deployment architecture.

Abstract

Information systems can retrieve records, connect entities and generate conclusions without establishing that an output preserves identity, remains connected to admissible evidence, respects the authority under which it was derived, or retains validity when moved between contexts. This paper introduces Governed Relational Geometry as an implementation-neutral formal model for those conditions. The model defines a governed relational geometry as a policy-indexed structure of positions, relationships, contexts and partial transformations. A governed position records identity conditions, admitted evidence, evaluation context, authority, material constraints, status and provenance lineage. A governed transformation maps input positions to an output while preserving or explicitly accounting for the properties required by policy.

The model separates content equivalence from governed equivalence; distinguishes declared context from evaluation context; separates evidentiary, validation and lifecycle states; and defines authority boundedness, constraint handling, contextual transport, contradiction containment, noninterference under irrelevant change and responsiveness to material change. A loss record makes permitted information loss explicit. Under stated compatibility and monotonicity assumptions, valid transformations compose; provenance continuity is proved by induction over finite transformation chains.

The framework complements provenance standards, knowledge graphs, argumentation, belief revision and contextual information-flow models by treating provenance, context, authority, constraints and scoped validation as jointly operative conditions on transformation validity. It supplies the formal layer between the field definition in Paper 1 and the external Machine Entity Comprehension conformance profiles planned for Paper 3.

Keywords: Geometry Intelligence; governed relationships; evidence; context; provenance; authority; constraint handling; valid transformation; machine entity comprehension; AI assurance.

1. Introduction

1.1 The structural problem

An information system may produce a plausible statement for the wrong entity, from inadmissible evidence, under expired authority, outside the relevant jurisdiction, or after removing a condition on which the statement depended. The words can remain unchanged while their governed meaning changes.

Let claim c be supported in declared context γ_1. Copying the same content into evaluation context γ_2 does not establish support in γ_2:

$$\text{Supported}(c, \gamma_1, \Pi) \not\Rightarrow \text{Supported}(c, \gamma_2, \Pi).$$

The transfer requires a justified mapping between contexts, evidence rules, authority scopes and material constraints. Without that mapping, the system has copied content rather than transported validity.

This distinction matters wherever a conclusion carries institutional consequences: legal reasoning, engineering assurance, formal mathematics, scientific evidence, policy, asset intelligence and sovereign information systems. In each domain, an outcome is governed by more than text. It is governed by the identity of its subject, evidence admitted, context of use, applicable authority, preserved constraints and validation procedure.

1.2 Research question

Under what public, implementation-neutral conditions may a transformation from evidence and claims to a derived outcome be treated as governed and valid?

The answer developed here is relational and scoped. Validity is not an intrinsic property of an isolated output. It is a property of an output position under a declared policy and evaluation context, connected to the transformation path that produced it.

1.3 Why geometry

Geometry is used in a generalized structural sense. The framework contains positions, typed adjacency, neighborhoods, boundaries, maps between positions and invariants under transformation. It does not assume Euclidean distance, continuity, a differentiable manifold or a proprietary dimensional representation. The relevant geometry is the governed structure of knowledge and its permissible movement.

1.4 Contributions

1. A policy-indexed mathematical structure for governed relational reasoning.

2. A typed object model for entities, representations, claims, evidence, contexts, authorities, constraints, statuses and lineage.

3. A formal distinction between content equivalence and governed equivalence.

4. A context model separating declared context from evaluation context and requiring explicit transport across material boundaries.

5. A model of valid transformation with explicit loss, constraint handling and authority boundedness.

6. A non-circular account of material change based on policy-relevant dependency.

7. A contradiction model separating representational, temporal, contextual, evidentiary, authority, policy and logical conflict.

8. Composition conditions and an inductive proof of provenance continuity.

9. A complete synthetic transformation trace and counterexample.

10. A bridge from Geometry Intelligence foundations to external conformance testing.

1.5 Scope

The paper specifies public semantics and proof obligations. It does not prescribe an implementation-specific coordinate representation, metric, storage model, learning architecture, algorithm or deployment design.

2. Relationship to the series

Paper 1 established the constitutional layer of Geometry Intelligence: the category definition, objects, principles, boundaries and hierarchy connecting Geometry Intelligence, Machine Entity Comprehension and sovereign Web4 systems [1]. Paper 2 formalizes the conditions attached to governed positions and transformations. Paper 3 will translate these conditions into black-box evaluation profiles.

$$\text{Paper 1: field definition} \rightarrow \text{Paper 2: public formalism} \rightarrow \text{Paper 3: external conformance.}$$

The present model refines rather than replaces Paper 1. Terms shared with the Jacobs Web4 Internet-Drafts retain their bounded meanings; those documents remain individual works in progress rather than standards [2–4].

3. Related work and contribution boundary

3.1 Provenance and lineage

The W3C PROV family supplies a domain-independent model of entities, activities and agents, including derivation, attribution and generation [5,6]. Database provenance and semiring approaches formalize how output annotations depend on input annotations [15]. Governed Relational Geometry accepts provenance as foundational but treats it as one condition among several: a derivation may be traceable while still crossing an unauthorized context boundary or exceeding its authority.

3.2 Knowledge graphs and contextualized statements

Knowledge graphs represent assertions among entities and increasingly support provenance, temporal qualification and contextualization [8,11]. Named graphs associate statements with graph-level identity, provenance and trust [18], while RDF-star permits statements about statements [17]. The present framework specifies when an edge or path may carry governed status under evidence, authority, context, constraints and validation.

3.3 Argumentation, inconsistency and belief change

Abstract argumentation formalizes attack and acceptability among arguments [12]. AGM belief revision formalizes rational change to belief sets [13]. Four-valued and paraconsistent approaches distinguish support, contradiction, both and neither without explosion [14]. These traditions inform the treatment of contradiction and state, while this model additionally preserves source authority, context transport, material constraints and transformation lineage.

3.4 Context and policy

Contextual integrity evaluates information flows relative to actors, information types, contexts and transmission principles [9,10]. Formal work on context emphasizes that assertions may hold relative to one context and require bridge rules for transfer [16]. Policy logics distinguish possession of information from permission to use or derive it [19].

3.5 Geometric and representation learning

Geometric deep learning exploits graphs, groups, symmetries and manifolds in learning architectures [7]. Governed Relational Geometry operates at a different level: it provides public acceptance conditions for transformations regardless of whether the implementation is vector, symbolic, graph-based, rule-based or hybrid.

3.6 Specific contribution

Governed Relational Geometry proposes a specific synthesis in which provenance, context, authority, constraints and scoped validation are jointly operative conditions on transformation validity. Its contribution lies in the policy-indexed structure, preservation and loss conditions, composition obligations and connection to external machine-comprehension evaluation—not in a claim that no prior system has combined related ideas.

4. Formal setting

4.1 Primitive domains

Symbol	Domain
$\mathcal{O}$	Informational objects
$\mathcal{N}$	Entities
$\mathcal{Q}$	Claims
$\mathcal{D}$	Evidence items
Γ	Contexts
$\mathcal{A}$	Authorities
$\mathcal{K}$	Constraints
$\mathcal{S}^{E}$	Evidentiary states
$\mathcal{S}^{V}$	Validation states
$\mathcal{S}^{L}$	Lifecycle states
$\mathscr{R}$	Governed relationships
$\mathcal{T}$	Transformations
$\mathfrak{P}$	Governing policies
$\mathscr{L}$	Provenance lineages
$\mathcal{X}$	Loss records
$\mathcal{P}$	Governed positions

An informational object may instantiate more than one role. A proof artifact may be evidence for one claim and an object of validation in another process.

4.2 Policy-indexed geometry

For policy $\Pi \in \mathfrak{P}$, define a governed relational geometry:

$$\mathfrak{G}\Pi = (\mathcal{P}, \mathcal{R}, \Gamma, \mathcal{T}, \preccurlyeq, \equiv g, \mathrm{Adm}\Pi, \mathrm{Valid}\Pi).$$

Here $\preccurlyeq$ is context refinement, $\equiv$g is governed equivalence, AdmΠ is admissibility and ValidΠ is scoped validity.

4.3 Entities and representations

For entity $n \in \mathcal{N}$, let $\rho(n) \subseteq \mathcal{O}$ be its associated representations. Association is itself governed:

$$\mathrm{Represents}\Pi(o, n, \gamma, d, a, k, s).$$

Representation equality is neither necessary nor sufficient for entity identity.

4.4 Claims

$$q = \langle \sigma, \pi, \omega, \gamma\mathrm{declq} \rangle,$$

where σ is subject, π predicate, ω asserted state and γdeclq the context declared by the source. Evaluation occurs under potentially different context γevalq.

4.5 Evidence

$$E(d) = \langle src, acq, int, scope, adm\rangle,$$

where src is source identity, acq acquisition information, int integrity information, scope evidentiary scope and adm admissibility under policy.

4.6 Context

$$\gamma = \langle t, j, i, g, u\rangle,$$

where t is temporal scope, j jurisdiction, i institutional setting, g domain or governing regime and u authorized use. Policy Π is kept separate.

4.7 Authority and constraints

$$a = \langle actor, domain, scope, interval, basis\rangle.$$

Authority is scoped rather than scalar. A constraint $k \in \mathcal{K}$ is a predicate over positions or transformations:

$$k : \mathcal{P} \to \{\top, \bot\} \quad \text{or} \quad k : \mathcal{T} \to \{\top, \bot\}.$$

4.8 Product state

$$S(o) = \langle S^E(o), S^V(o), S^L(o)\rangle,$$

Axis	Illustrative values
S^E — evidentiary	unsupported; supported; contested
S^V — validation	unexamined; indeterminate; accepted; rejected
S^L — lifecycle	current; superseded; withdrawn

5. Governed positions and relationships

5.1 Governed position

$$P\Pi(o, \gamma eval) = \langle id(o), Eo, \gamma decl, \gamma eval, Ro, Ao, Ko, S(o), Lo\rangle.$$

The components identify identity condition, admitted evidence, declared and evaluation contexts, relationship neighborhood, authorities, material constraints, product state and provenance lineage.

5.2 Declared completeness

$$Complete\Pi(P) = \top$$

when every component required by policy is present or explicitly unavailable. Completeness is inspectability, not correctness.

5.3 Content and governed equivalence

Content equivalence $P_1 \equiv c\ P_2$ means that asserted propositions are semantically equivalent. Governed equivalence $P_1 \equiv g,\Pi,u\ P_2$ additionally requires policy-relevant equivalence of identity, evidence, contexts, authority, constraints, state and required lineage.

$$P_1 \equiv g,\Pi,u\ P_2 \;\Rightarrow\; P_1 \equiv c\ P_2,$$

$$P_1 \equiv c\ P_2 \;\not\Rightarrow\; P_1 \equiv g,\Pi,u\ P_2.$$

5.4 Governed relationship

$$r = \langle \mathrm{src}, \tau, \mathrm{tgt}, \gamma r, \mathrm{Er}, \mathrm{Ar}, \mathrm{Kr}, \mathrm{S}(r), \mathrm{Lr} \rangle.$$

Relationship types may include supports, contradicts, derives-from, depends-on, supersedes, governs, applies-within, authorizes, invalidates and equivalent-under.

5.5 Admissibility and validity

$$\mathrm{Adm}\Pi(r, \gamma) \in \{\top, \bot, ?\}.$$

$$\mathrm{ValidRel}\Pi(r, \gamma) \Leftrightarrow \mathrm{ContextOK} \wedge \mathrm{EvidenceOK} \wedge \mathrm{AuthorityOK} \wedge \mathrm{ConstraintsOK} \wedge \mathrm{ProvenanceOK} \wedge \mathrm{StateOK}.$$

6. Context boundaries and transport

6.1 Refinement and compatibility

Write $\gamma_1 \preccurlyeq \gamma_2$ when γ_1 refines γ_2 for dimensions relevant to the operation.

$$\mathrm{Compat}\Pi\mathrm{,u}(\gamma_1, \gamma_2) \in \{\top, \bot, ?\}.$$

6.2 Boundary and bridge rule

$$\mathrm{Boundary}\Pi\mathrm{,u}(\gamma_1, \gamma_2) = \top$$

when movement changes a policy-relevant dimension. Transport requires:

$$\mathrm{F}\gamma_1{\rightarrow}\gamma_2 : \mathcal{P}\gamma_1 \rightharpoonup \mathcal{P}\gamma_2.$$

6.3 Loss record

$$\Lambda\mathrm{F} = \langle \text{lost evidence, lost authority, lost constraints, lost precision, unresolved conditions} \rangle \in \mathcal{X}.$$

A loss record may be empty. Silent loss of policy-material structure invalidates a transformation.

7. Transformations

7.1 Definition

$$\mathrm{T} : (\mathcal{P}^n \times \Gamma \times \mathfrak{P}) \rightharpoonup (\mathcal{P} \times \mathcal{X} \times \mathcal{L}$$

7.2 Classes

1. Identity normalization: maps equivalent representations to one entity position.

2. Evidence admission: admits evidence under policy.

3. Evidence withdrawal: removes or invalidates evidence while retaining history.

4. Context transport: crosses a declared context boundary.

5. Derivation: produces a claim from admitted premises.

6. Aggregation: combines positions while declaring reduced granularity.

7. Adjudication: scopes or resolves conflict under authority.

8. Supersession: replaces current status while preserving lineage.

9. Redaction: removes information under an authorized loss record.

10. Validation: changes validation status through a declared procedure.

7.3 Transformation record

$$\mathrm{Rec}(T) = \langle \mathrm{idT}, \mathrm{inputs}, \mathrm{output}, \mathrm{actor}, \mathrm{time}, \mathrm{policy}, \mathrm{context}, \mathrm{loss}, \mathrm{procedure} \rangle.$$

7.4 Constraint handling

$$\mathrm{ConstraintsHandled}\Pi(T) \Leftrightarrow \mathrm{ConstraintsPreserved}(T) \lor (\mathrm{LossPermitted}\Pi(T) \land \mathrm{LossDeclared}(T)).$$

7.5 Authority boundedness

$$\mathrm{Scope}(AP') \subseteq \mathrm{Grant}\Pi(AP_1, \ldots, AP_n, T).$$

Aggregation alone grants no new authority. Delegated procedures may create bounded authority only where policy grants it.

7.6 Valid transformation

$$\mathrm{ValidT}\Pi(T) \Leftrightarrow \mathrm{Defined} \land \mathrm{InputsAdmissible} \land \mathrm{IdentityPreserved} \land \mathrm{ProvenancePreserved} \land \mathrm{ConstraintsHandled} \land \mathrm{AuthorityBounded} \land \mathrm{ContextAuthorized} \land \mathrm{LossDeclared} \land \mathrm{OutputWellFormed}.$$

$$\mathrm{ValidatedT}\Pi(T) \Leftrightarrow \mathrm{ValidT}\Pi(T) \land \mathrm{ValidationAccepted}\Pi(P').$$

8. Axioms and invariants

Axiom	Statement
A1 — Scoped position	Every governed claim has the components required by policy.
A2 — Representation separation	Representation equality is neither necessary nor sufficient for entity identity.
A3 — Provenance requirement	Every material derived position retains required lineage.
A4 — Constraint accountability	Every material constraint is preserved or recorded as authorized loss.
A5 — Authority boundedness	Transformation authority is bounded by explicit policy grant.
A6 — Context-bound validity	Validation does not cross contexts without an authorized bridge.
A7 — Contradiction persistence	Conflict remains represented until authorized adjudication.
A8 — Material responsiveness	A material change affects dependent outputs.
A9 — Irrelevant noninterference	An irrelevant change does not alter material outcome.
A10 — Declared loss	Concealed material loss invalidates transformation.

8.1 Identity preservation

$$\mathrm{Id}(P') \equiv \mathrm{IdT}(P_1, \ldots, P_n).$$

8.2 Provenance continuity

$$L(P') \supseteq \mathrm{RequiredLineage}\Pi(P_1, \ldots, P_n) \cup \{\mathrm{Rec}(T)\}.$$

8.3 Constraint preservation

$$P' \models \mathrm{PushT}(\mathrm{Kmat})$$

for material constraints required to survive. PushT must be monotone for composition.

8.4 Scoped validation

$$V(P') = \langle \mathrm{state}, \gamma\mathrm{eval}, \Pi, \mathrm{procedure}, \mathrm{time}, \mathrm{version} \rangle.$$

9. Change, dependency and materiality

9.1 Dependency

$$\mathrm{Dep}\Pi(x, y) = \top$$

when the governed status of y depends on x under policy Π.

9.2 Perturbation

$$\delta : \mathcal{P} \to \mathcal{P}\delta.$$

$$\mathrm{Material}\Pi(\delta, x, y) \Leftrightarrow \mathrm{Dep}\Pi(x, y) \wedge \mathrm{RelevantPredicateChanged}\Pi(\delta, x).$$

9.3 Noninterference and responsiveness

$$\delta \in \Delta i \;\Rightarrow\; f\Pi(P) \equiv g,\Pi,u\; f\Pi(\delta(P)).$$

10. Contradiction and adjudication

Difference is not contradiction. Direct contradiction requires aligned entity identity, materially comparable context and incompatible asserted states:

$$\mathrm{Contradicts}\Pi(q_1, q_2) \Leftrightarrow \mathrm{SameEntity}(\sigma_1, \sigma_2) \wedge \mathrm{ContextComparable}\Pi(\gamma_1, \gamma_2) \wedge \mathrm{Incompatible}\Pi(\pi_1, \omega_1, \pi_2, \omega_2).$$

Class	Meaning
Representational difference	Different wording or identifiers.
Temporal difference	Assertions apply at different times.
Contextual difference	Assertions apply under different contexts.
Evidentiary conflict	Evidence supports incompatible outcomes.
Authority conflict	Authorities issue incompatible positions.
Policy conflict	Policies permit incompatible outcomes.
Logical contradiction	Assertions cannot jointly hold under one formal regime.

Containment retains both claims, evidence lineages, conflict class and pre-adjudication state.

$$J : (q_1, q_2, E, A, \gamma, \Pi) \rightharpoonup (PJ, \Lambda J, LJ).$$

11. Composition properties

11.1 Interface compatibility

Transformations $T_1 : P_0 \rightharpoonup P_1$ and $T_2 : P_1 \rightharpoonup P_2$ are compatible when the output context, state, constraints, authority and lineage of T_1 satisfy the input requirements of T_2. Policies must be identical or connected by an explicit compatibility map.

11.2 Theorem 1 — composition under preservation

Theorem. Let T_1 and T_2 be valid, interface-compatible transformations. Assume identity mappings compose, lineage inclusion is transitive, constraint pushforwards are monotone, authority grants are compositional, context bridges compose, and losses are declared. Then $T_2 \circ T_1$ is valid.

Proof. Definedness and input admissibility follow from interface compatibility. Identity preservation follows by composition. Provenance continuity follows because T_2 receives the required lineage of T_1 and appends its own record. Monotonicity preserves surviving material constraints; permitted losses remain declared. Authority boundedness follows from nested policy grants. Composable bridge rules authorize context movement. Output well-formedness follows from validity of T_2. Therefore every conjunct of ValidTΠ holds. ■

11.3 Theorem 2 — provenance continuity over finite chains

Theorem. Let P_0 have valid lineage. For finite chain $T_1, \ldots, T_n$, suppose every T_i preserves required lineage and appends $\mathrm{Rec}(T_i)$. Then every P_i has continuous provenance:

$$L(P_n) \supseteq \mathrm{RequiredLineage}\Pi(P_0) \cup \{\mathrm{Rec}(T_1), \ldots, \mathrm{Rec}(T_n)\}.$$

Proof by induction. The base case holds by assumption. If P_n has continuous lineage, T_{n+1} preserves it and appends $\mathrm{Rec}(T_{n+1})$; therefore P_{n+1} contains the initial lineage and all records through $n + 1$. ■

11.4 Corollaries and counterexample

Authority corollary. Without an independent grant, aggregation output cannot exceed the permitted combination of input scopes. Silent-loss corollary. Removing a material constraint without permitted and declared loss is invalid by A4 and A10.

Content-equivalence counterexample. Two positions contain the identical statement "Contract C belongs to Orion Infrastructure." P_1 is evaluated in Jurisdiction X under authority A_x; P_2 is evaluated in Jurisdiction Y without an authorized bridge. Then $P_1 \equiv c\ P_2$ but $P_1 \not\equiv g\ P_2$.

12. Worked transformation

12.1 Scenario

Two similarly named organizations operate in different jurisdictions. Procurement record d_1 attributes contract c to n_x. Commercial record d_2 attributes it to n_γ. Later correction d_3 identifies d_2 as erroneous.

Object	Governed content
n_x	Orion Infrastructure Holdings Ltd; Jurisdiction X; identifier X-441.
n_γ	Orion Infrastructure Holding LLC; Jurisdiction Y; identifier Y-908.
q_1	n_x holds contract c under γ_x.
q_2	n_γ holds contract c under γ_γ.
d_1	Authoritative procurement record supporting q_1.
d_2	Commercial record supporting q_2.
d_3	Authoritative correction identifying d_2 as an attribution error.

12.2 Initial positions

$$P(q_1) = \langle n_x, \{d_1\}, \gamma_x\text{decl}, \gamma_x\text{eval}, R_1, \{\text{Aproc}\}, K_1, \langle\text{supported, unexamined, current}\rangle, L_1\rangle.$$

$$P(q_2) = \langle n_\gamma, \{d_2\}, \gamma_\gamma\text{decl}, \gamma_x\text{eval}, R_2, \{\text{Acommercial}\}, K_2, \langle\text{supported, unexamined, current}\rangle, L_2\rangle.$$

12.3 Transformation trace

Step	Transformation	Result	Loss
T_1	Identity normalization	Preserves n_x and n_γ as distinct.	∅
T_2	Evidence admission	Admits d_1 as authoritative and d_2 as secondary.	∅
T_3	Conflict classification	Classifies attribution conflict, not direct logical contradiction.	∅
T_4	Bounded derivation	q_1 supported under γ_x; q_2 remains conflicting secondary attribution.	Precision bounded to γ_x.
T_5	Supersession after d_3	Marks q_2 superseded while preserving lineage.	∅

12.4 Material and irrelevant changes

Capitalization changes no policy-relevant identity predicate and is irrelevant. Correction d_3 changes admissibility and lifecycle status and is material. Affected output must change while unrelated relationships remain stable.

12.5 Context transport

Using the procurement conclusion in Jurisdiction Y requires a bridge rule. Copying without the rule preserves content but not governed validity. Permitted transport records changed authority, constraints and unresolved conditions in ΛF.

13. Domain-specific validation

Domain	Illustrative validation regime
Formal mathematics	Axioms, proof object, checker and library version.
Law	Authority, jurisdiction, effective date, procedure and admissibility.
Engineering	Requirements, tolerances, versions, assumptions and tests.
Scientific evidence	Study design, measurement, method, replication and review.
Institutional decision	Delegated authority, policy version, evidence and approval.

$$\mathrm{ValidDomain}(P, \gamma, \Pi, \mathrm{procedure}, \mathrm{version}).$$

No universal scalar substitutes for the applicable validation regime.

14. Implementation and conformance implications

An implementation may contain identity, evidence, context, relationship, constraint, authority, transformation, validation, audit and federation layers. It may use databases, graphs, formal logic, event logs, learned models or hybrid systems. Conformance depends on behavior and inspectability rather than a technology label.

Property	Observable implication for Paper 3
Identity persistence	Equivalent representations preserve identity.
Entity disambiguation	Similar representations of distinct entities remain separate.
Relationship invariance	Irrelevant perturbations preserve governed outcomes.
Material responsiveness	Material changes alter affected outcomes.
Contradiction containment	Conflicting assertions remain attributable.
Repeatability	Equivalent state produces governed-equivalent outcomes.
Multi-node equivalence	Equivalent authorized nodes produce equivalent tested outcomes.
Disconnected continuity	Declared core capability remains without undeclared external dependencies.

15. Security and governance

Threat	Governed failure
Entity substitution	Identity is merged, replaced or redirected.
Evidence poisoning	False or context-stripped material is admitted.
Provenance forgery	A derivation path is falsely asserted.
Authority spoofing	An actor impersonates authority or exceeds scope.
Context laundering	A statement moves without declared condition changes.
Constraint suppression	Material assumptions are removed.
Adjudication capture	Resolving authority or policy is manipulated.
Validation detachment	A result is separated from procedure, policy or version.

Auditability does not require unrestricted publication. Confidential lineage may be access-controlled, but an authorized evaluator must be able to inspect it. Correction and contestation should preserve historical state rather than silently erase it.

16. Limitations and falsifiability

- The framework is formal but abstract; domain predicates require domain semantics.
- A governed output may still be wrong if evidence, policy or validation is flawed.
- The framework defines no universal metric, score or distance.
- Equivalent behavior may arise from different implementations.
- The model does not eliminate authorized judgment.
- The model does not establish consciousness or human-equivalent understanding.
- Composition results hold only under stated assumptions.
- Independent empirical validation remains future work.

The framework is falsifiable or restrictable through counterexamples showing an invariant unnecessary, insufficient or inconsistent; domains where scoped grant cannot model authority; failures of local-to-global composition; or simpler models reproducing the same conformance behavior.

17. Research programme

1. Define denotational semantics for positions, relationships, policies and loss records.

2. Develop an algebra of constraint composition, weakening, incompatibility and pushforward.

3. Formalize delegation, expiry, conflict and grant composition for authority.

4. Define faithful, lossy and prohibited context transports.

5. Develop machine-evaluable conflict classes using multi-valued or paraconsistent semantics.

6. Encode the public structures and selected theorems in Lean 4.

7. Translate invariants into Machine Entity Comprehension conformance profiles.

8. Compare governed transformation with retrieval-only, graph-only, rule-only and generative baselines.

9. Support an independent implementation that does not use KTS methods.

10. Conduct domain studies in law, engineering, formal mathematics and institutional decision systems.

18. Conclusion

Governed Relational Geometry formalizes a central proposition of Geometry Intelligence: the validity of a machine outcome depends not only on content, but on the governed structure connecting identity, evidence, declared and evaluation contexts, authority, constraints, state and transformation lineage.

A conclusion may preserve its words while losing validity. A relationship may exist while lacking admissible evidence. A derivation may be coherent while exceeding authority. A summary may silently discard a material condition.

The model uses a policy-indexed geometry of positions, relationships, boundaries and partial transformations. It requires identity preservation, provenance continuity, constraint handling, authority boundedness, authorized context transport, declared loss and scoped validation.

A machine outcome is governed only when the path from evidence to conclusion preserves—or explicitly and permissibly accounts for—the identity, provenance, authority, context and material constraints required for that conclusion to retain its scoped validity.

Paper 3 will turn these invariants and failure conditions into externally observable conformance profiles for Machine Entity Comprehension.

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Appendix A — Notation

Symbol	Meaning
$\mathfrak{G}\Pi$	Governed relational geometry under policy Π
$\mathcal{P}$	Governed positions
$\mathscr{R}$	Governed relationships
Γ	Contexts
$\mathcal{T}$	Transformations
$\mathfrak{P}$	Policies
$\mathscr{L}$	Lineages
$\mathcal{X}$	Loss records
$P\Pi(o, \gamma\mathrm{eval})$	Governed position in evaluation context
$\gamma\mathrm{decl}$	Context declared by source
$\gamma\mathrm{eval}$	Context of evaluation
ΛT	Transformation loss record
$\equiv c$	Content equivalence
$\equiv g,\Pi,u$	Governed equivalence
$\preccurlyeq$	Context refinement
$\mathrm{Dep}\Pi$	Policy-relevant dependency
$\mathrm{Adm}\Pi$	Admissibility
$\mathrm{Valid}T\Pi$	Valid transformation
$\mathrm{Validated}T\Pi$	Validated transformation

Appendix B — Suggested citation

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